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Probability of Failure and Risk Assessment of Propulsion Structural Components

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PROBABILITY OF FAILURE AND RISK ASSESSMENT OF PROPULSION

STRUCTURAL COMPONENTS

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SUMMARY

Because of the increasing need to account for the uncertainties in material properties, loading conditions, geometry, etc., a methodology has been developed to determine structural reliability and to assess the associated risk. The methodology consists of a probabilistic structural analysis by a probabilistic finite element computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) and a generic probabilistic material property model. The methodology is versatile and is equally applicable to structures operating at high- and cryogenic-temperature environments. Results obtained demonstrate that the issues of structural reliability and risk can be formally evaluated by using the methodology developed which is inclusive of the uncertainties in material properties, structural parameters, and loading conditions. The methodology is described in some detail with illustrative examples.

INTRODUCTION

The probabilistic structural analysis methods (PSAM) (ref. 1) have been developed to analyze the effects of fluctuating loads, variable material properties, uncertainties in analytical models and geometry, etc., especially for high-performance structures such as space shuttle main engine (SSME) turbopump blades. In the deterministic approach, uncertainties in the responses are not quantified, and the actual safety margin remains unknown. Risk is calculated after extensive service experience. However, probabilistic structural analysis provides a rational alternative method to quantify uncertainties in structural performance and durability. NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) is a probabilistic structural analysis computer code developed under the PSAM project which integrates finite element methods and reliability algorithms (refs. 2 and 3). This code is capable of predicting the scatters of structural response variables (such as stress, displacement, natural frequencies, and buckling loads) subjected to all kinds of uncertainties. These are subsequently compared with their probable failure modes to assess the risk of component fracture. Probable failure modes are defined for different structures and their respective service environments. For example, failure events (such as stress greater than strength, displacement exceeds

E-5019

maximum allowables, or avoidance of resonance) are often used for the reliability assessment. Probability of occurrence of those failure events can be determined once the probability distributions of the requisite structural response variables are calculated by NESSUS.

In undertaking a reliability and risk analysis, all suspected sources of uncertainties must be taken into account in order to control the probability of failure in service environments within an acceptable range. Structural reliability and risk obtained by a formal probabilistic methodology can be useful in evaluating the traditional design and in setting quality control requirements, inspection criteria, and retirement for cause. This methodology can also be used to identify candidate materials and design concepts in the absence of a technology base. In this report, the methodology developed to assess the structural reliability and risk/cost is described.

CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS

In a probabilistic structural analysis, the primitive variables which affect the structural behaviors have to be identified. These include temperature, material properties, structural geometry, loading conditions, etc. and should be described by their respective probability distributions. A structural analysis performed by NESSUS with the predetermined probability distributions of all the primitive (random) variables will produce corresponding scatter (uncertainties) in the structural responses such as displacement, stress, natural frequencies, etc. The concept is illustrated by figure 1.

STRUCTURE OF NESSUS

NESSUS consists of three major modules: NESSUS/PRE, NESSUS/FEM, and NESSUS/FPI. NESSUS/PRE is a preprocessor used for the preparation of the statistical data needed to perform the probabilistic finite element analysis. It allows the user to describe the uncertainties in the structural parameters (random variables) at nodal points of a finite element mesh. The uncertainties in these parameters are specified over this mesh by defining the mean value and standard deviation of the random variable at each point, together with an appropriate form of correlation. Correlated random variables are then decomposed into a set of uncorrelated vectors by a modal analysis. For a strongly correlated random field, the number of dominant random variables in the set of uncorrelated vectors will be much less than the number in a weakly correlated random field. Thus, the computational time required for the analysis will also be reduced significantly.

NESSUS/FEM is a finite element code used for the structural analysis and parameter sensitivity evaluation. It generates a database containing all the response information corresponding to a small variation of each independent random variable. The algorithm used in NESSUS/FPI requires an explicit response function in order to perform a reliability analysis. In complicated structural analysis problems, response can only be available implicitly through a finite element model. To overcome this difficulty, the response function is expressed parametrically with the database.

NESSUS/FPI (Fast Probability Integrator) is an advance reliability module (ref. 3). This module extracts the database generated by NESSUS/FEM to develop a response or a performance model in terms of uncorrelated random variables. The probabilistic structural response is calculated from the performance model. For a given response value, the probability of exceeding this value is estimated by a reliability method, which treats the problem as a constrained minimization. This step is called a point probability estimation. The cumulative distribution function is generated by running FPI at several response values. One alternative for generating the distribution function for any given response is to conduct a direct Monte Carlo simulation study. However, in general, it is very costly. NESSUS/FPI provides a method which not only produces an accurate probability distribution, but also requires less computing time than that required for Monte Carlo simulation, especially in low-probability regions.

PROBABILISTIC MATERIAL PROPERTIES MODEL

A generic material behavior model (ref. 4) is used to evaluate the scatters of material properties for the structures subjected to high-temperature environments and high-cycle loading conditions. The fundamental assumption for this model is that the material properties behavior can be simulated by primitive (random) variables. The general form of this model is shown in equation (1).

$$M_P = M_{PO}\left[\frac{T_F - T}{T_F - T_O}\right]^n \left[\frac{S_F - \sigma}{S_F - \sigma_O}\right]^p \left[\frac{\log N_{MF} - \log N_M}{\log N_{MF} - \log N_{MO}}\right]^q \tag{1}$$

where M_P is the material property at current temperature after the application of fatigue cycles; M_{PO} is the reference material property at reference temperature T_O (usually room temperature), reference stress σ_O, and reference fatigue cycles N_{MO}; T_F, S_F, and N_{MF} are the final temperature, final material strength, and final mechanical cycle, respectively; T is current blade temperature; σ is the existing blade stress; and N_M is the number of fatigue cycles to be considered in the failure analysis. The exponents n, p, and q are determined from available experiment data, or they are estimated from the anticipated material behavior due to the particular primitive random variable.

APPLICATION TO A SPACE SHUTTLE MAIN ENGINE (SSME) TURBINE BLADE

The methodology described previously was applied to a SSME turbine blade (fig. 2) which was subjected to complex mechanical and thermal loads. (Probabilistic Structural Analysis for Select Space Propulsion System Components. First Annual Report by Southwest Research Institute, Rocketdyne, University of Arizona, Marc Analysis Corp., and Columbia University for NASA Lewis Research Center under Contract NAS3-24389, March 1986.) Mechanical loads consist of centrifugal force and the differential pressure across the airfoil. Centrifugal force is induced by rotational speed. Since it is difficult to maintain a constant rotational speed, the centrifugal force has to be considered as a random variable. Differential pressure is also random because of pressure fluctuation. Random thermal loads are due to combustion irregularities which cause a random temperature distribution in the blade. Uncertainties in the blade geometry arise during the manufacturing process.

3

The scatter in material properties is caused by nonuniformities in the material.

The turbine blade is modelled by 40 4-node shell elements with 55 nodal points. In this study, seven random fields were considered as listed in table I. In previous analysis (ref. 5), the probability density function of material properties such as Young's modulus, thermal expansion coefficient, and material strength are assumed. In the present study, they are simulated by the probabilistic material property model as defined by equation (1). The material property is a function of its reference value, temperature, and fatigue cycles. The statistics of the primitive random variables are listed in table II. Since the material is a function of stress, and stress is a function of material property in an implicit way, an iterative procedure is necessary to obtain a convergent solution for stress and material properties.

To start the iteration procedure, the probability distributions of the material properties were predicted by equation (1) with its reference value and the blade temperature only, since the variations of the blade stresses were unknown. A probabilistic structural analysis using NESSUS was performed, and the probability distributions of stresses were determined. The generic probabilistic material property model was applied again at the given fatigue cycle with these newly estimated stress variations. The probability distributions of material properties were updated, and NESSUS was rerun. The procedure was repeated until the probability distributions of the material properties and stress converged. During the iteration process, the joint cumulative distribution functions of nodal stresses were also calculated in order to apply the generic probabilistic material property model on the entire blade. It was found that only a few iterations were needed for a convergence.

The blade is assumed to be subjected to 100 000 constant amplitude load cycles which degrade the modulus and strength. The critical points of the large displacement and high stress are depicted in figure 3. As shown in figure 4, at the root of the leading edge, the probability distribution of modulus is reduced significantly after the application of cyclic loads, where the initial distribution is that without the fatigue load, and the final distribution is the one after the application of the fatigue load. Changes in the probability density function of tip displacement with and without the application of cyclic loads are also shown in figure 5. As expected, the tip displacements are increased after the application of cyclic loads because the blade becomes softer. Material strength degradation after cyclic loads is calculated with the converged stress by the probabilistic material property model as shown in figure 6. Once the stress-strength relationships are determined at a given fatigue cycle, the probability that the stress is greater than the corresponding material strength will be determined. The probability is calculated by

$$P_f = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{x} f_S(s)ds \right] f_\sigma(x)dx \qquad (2)$$

where f_σ is the probability density function (pdf) of effective stress calculated by the probabilistic structural analysis using NESSUS, and f_S is the pdf of material strength simulated by the probabilistic material property model. By varying the number of cycles and repeating the procedure described previously, a Risk-Fatigue cycle curve is developed for critical locations as shown in figure 7. This curve is useful for assessing the risk of structural

4

fracture. For instance, at a given acceptable risk level, the number of fatigue cycles to initial local failure can be determined. With this information available, criteria can be set for quality control, inspection intervals, and retirement for cause.

The risk-cost assessment is the evaluation of the relationship between structural reliability and total cost associated with the structure. Total cost is the sum of initial cost and a fraction of consequential cost as defined in equation (3). The fraction is weighted by the probability of failure. The initial cost is the cost for component service readiness. The consequential cost is the cost incurred due to failure.

$$C_t = C_i + P_f \times C_f \tag{3}$$

where C_t represented total cost, C_i represented initial cost, P_f is the probability of failure, and C_f is the consequential cost. Since the lower initial cost is often associated with higher risk for the structural failure, and higher initial cost will normally reduce the risk, the total cost can be minimized for an acceptable structural reliability. For example, the initial cost is a function of the mean strength of the material as shown in figure 8. The greater the strength, the more reliable the structure. However, high initial cost is needed to improve the mean strength. Total cost is shown in figure 9. There exists a point of diminishing return. This can be identified as the lowest total cost with no loss of structural reliability. A similar result was also obtained for the case of strength quality improvement as shown in figures 10 and 11.

SUMMARY OF RESULTS

In summary, a reliability and risk cost methodology has been developed. It consists of a probabilistic structural analysis by NESSUS and a generic probabilistic material model. The methodology is versatile and equally applicable to hot and cold structures where data is difficult to obtain. The methodology is demonstrated by using it to assess the risk associated with fatigue cycles to initiate local failure in SSME blades.

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5. Shiao, M.C.; Nagpal, V.K.; and Chamis, C.C.: Probabilistic Structural
   Analysis of Aerospace Components Using NESSUS. 29th Stuctures, Structural
   Dynamics and Materials Conference, Part 3, AIAA, 1988, pp. 1275-1282.

TABLE I. - RANDOM INPUT DATA

Random fields	Number of dependent random variables	Mean	Standard deviation (or coefficient of variation	Correlation length	Number of independent random variables
X coordinate	55	Deterministic coordinate	0.01 in.	5.0	13
Y coordinate	55	Deterministic coordinate	0.01 in.	5.0	13
Z coordinate	55	Deterministic coordinate	0.01 in.	5.0	13
Temperature	55	Steady-state temperature	60 °F	3.0	22
Modulus	40	23 000 000 psi	0.10 (coefficient of variation)	3.0	16
Pressure	36	Steady-state pressure	0.20 (coefficient of variation)	0.0	36
Rotation speed	1	40 000 rpm	0.01 (coefficient of variation)	N/A	1

TABLE II. - PRIMATIVE VARIABLE PROBABILITY
DISTRIBUTIONS FOR PROBABILISTIC MATERIAL
PROPERTY MODEL

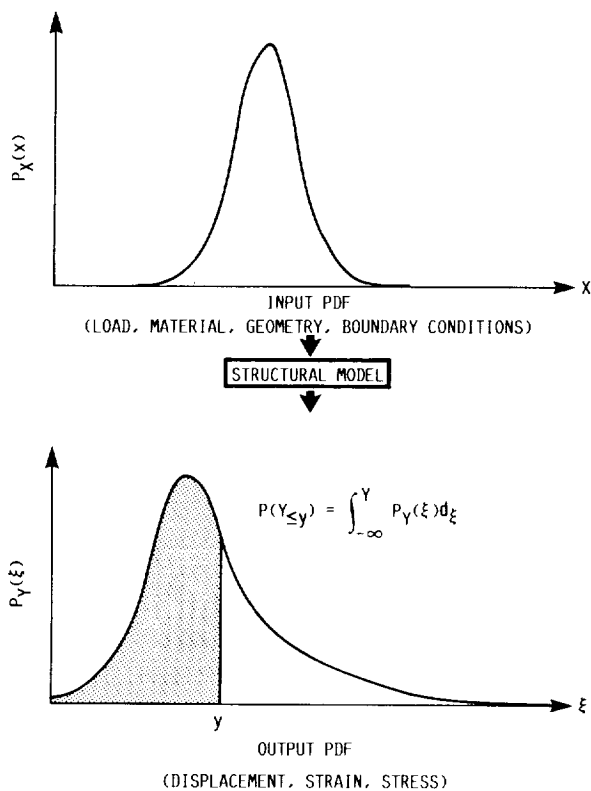| Variable	Distribution type	Mean	Standard deviation
T_F, °F	Normal	2750	51.4
T_0, °F	Normal	68	2.04
S_F, ksi	Normal	212.0	10.6
σ_0	Constant	0	0
N_{MF}	Lognormal	10^8	5×10^8
N_{MO}	Lognormal	10^3	50
n	Normal	0.25	0.0075
p	Normal	0.25	0.0075
q	Normal	0.25	0.0075

6

$$P(Y_{\leq y}) = \int_{-\infty}^{Y} P_Y(\xi)d\xi$$

INPUT PDF
(LOAD, MATERIAL, GEOMETRY, BOUNDARY CONDITIONS)

STRUCTURAL MODEL

OUTPUT PDF
(DISPLACEMENT, STRAIN, STRESS)

FIGURE 1. - CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS.

STOCHASTIC THERMOMECHANICAL LOADS

DYNAMIC RESPONSE

STOCHASTIC MATERIAL PROPERTIES

STRESS CONCENTRATION

UNCERTAIN BOUNDARY CONDITIONS

UNCERTAIN STRUCTURAL SHAPE
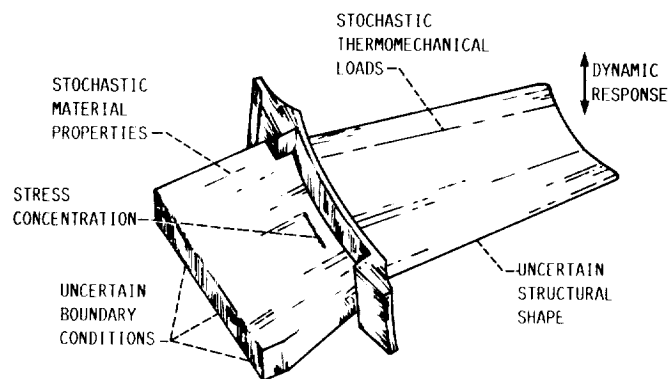
FIGURE 2. - UNCERTAINTIES IN PROBABILISTIC STRUCTURE ANALYSIS.

GLOBAL DISPLACEMENTS

LOCAL STRESSES
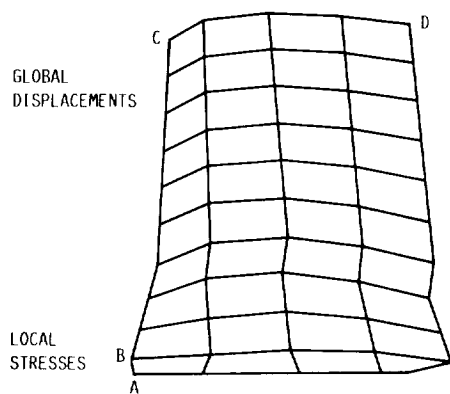
FIGURE 3. - SSME BLADE SHOWING LOCATIONS WHERE PROBABILISTIC STRUCTURAL RESPONSE WAS EVALUATED (RANDOM LOADINGS: PRESSURE, TEMPERATURE, AND ROTATIONAL SPEED; RANDOM GEOMETRY; RANDOM MATERIAL PROPERTIES; MECHANICAL CYCLE = 1000 000).
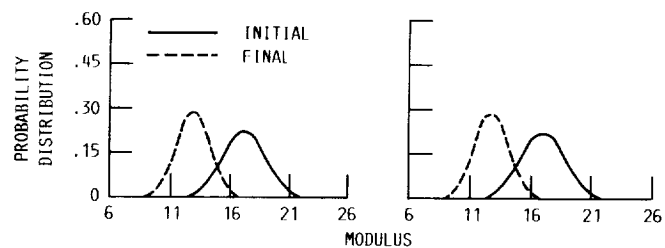
(A) NODE A. (MEAN STRESS STRENGTH RATIO = 0.83).

(B) NODE B. (MEAN STRESS STRENGTH RATIO = 0.82).

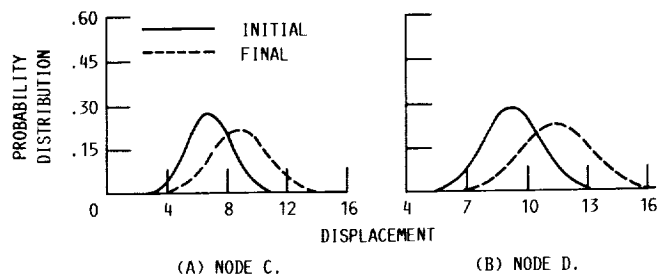FIGURE 4. - PROBABILISTIC MODULUS SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL — (MPSI).

PROBABILITY DISTRIBUTION

INITIAL
FINAL

.60
.45
.30
.15

0   4   8   12   16
DISPLACEMENT

(A) NODE C.

4   7   10   13   16

(B) NODE D.

FIGURE 5. - PROBABILISTIC DISPLACEMENTS CALCULATED BY
NESSUS ← (10^{-3} IN.).

PROBABILITY DISTRIBUTION

STRESS
STRENGTH

.100
.075
.050
.025
0

70   90   110   130   150
STRESS/STRENGTH

(A) NODE A.   (MEAN STRESS
STRENGTH RATIO = 0.83).

(B) NODE B.   (MEAN STRESS
STRENGTH RATIO = 0.82).
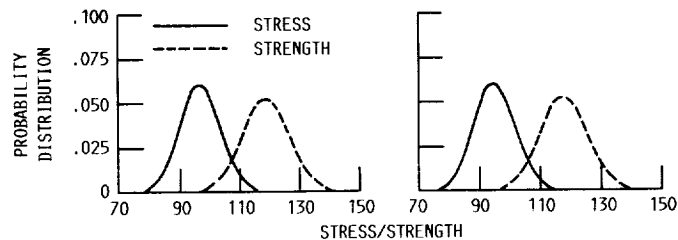
FIGURE 6. - PROBABILISTIC FATIGUE STRESS/STRENGTH SIMULATED
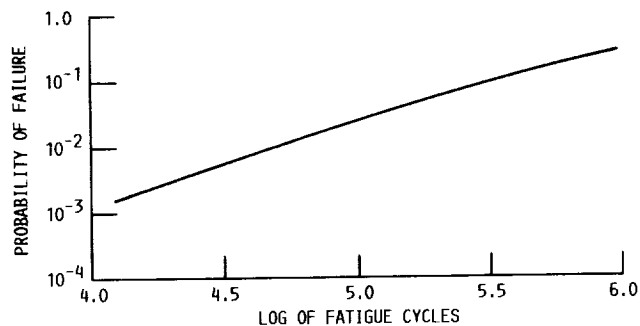BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL,
KSI.

PROBABILITY OF FAILURE

10^{-1}
10^{-2}
10^{-3}
10^{-4}

4.0   4.5   5.0   5.5   6.0
LOG OF FATIGUE CYCLES

FIGURE 7. - PROBABILITY OF LOCAL FAILURE DUE TO FATIGUE
CYCLES AT NODE A.

INITIAL COST   PROBABILITY OF FAILURE

O COST
□ PROBABILITY

5
4
3
2
1

110   120   130   140   150
MEAN STRENGTH

.300
.225
.150
.075
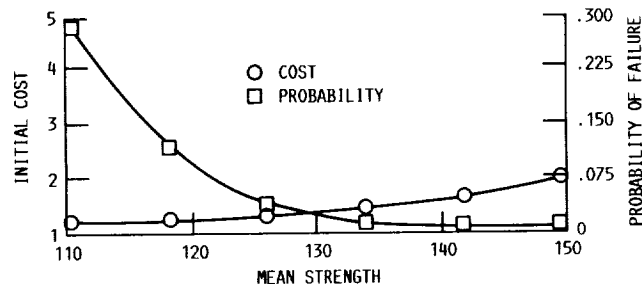
FIGURE 8. - INITIAL COST AND STRUCTURAL RELIABILITY
QUANTIFIED IN TERMS OF MEAN STRENGTH (GIVEN QUALITY).

TOTAL COST

5
4
3
2
1

110   120   130   140   150
MEAN STRENGTH
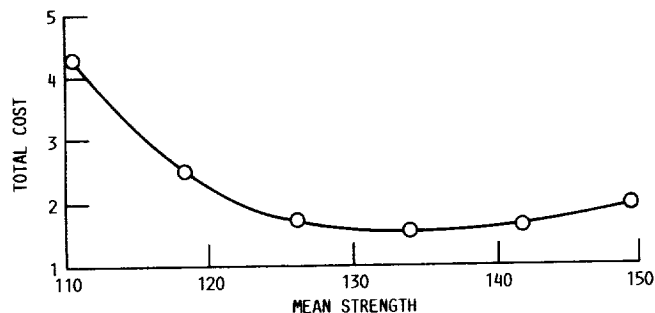
FIGURE 9. - TOTAL COST QUANTIFIED IN TERMS OF MEAN STRENGTH
(GIVEN QUALITY).

INITIAL COST   PROBABILITY OF FAILURE

O COST
□ PROBABILITY

2.5
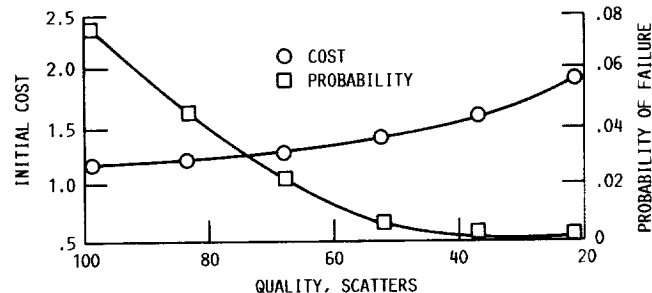2.0
1.5
1.0
.5

100   80   60   40   20
QUALITY, SCATTERS

.08
.06
.04
.02

FIGURE 10. - INITIAL COST AND STRUCTURAL RELIABILITY
QUANTIFIED IN TERMS OF QUALITY CONTROL (GIVEN MEAN
STRENGTH).

TOTAL COST

2.5
2.0
1.5
1.0
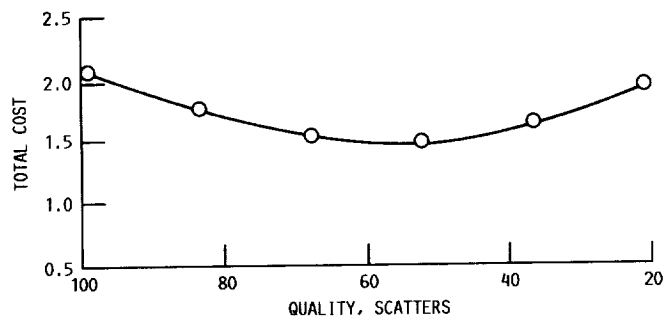0.5

100   80   60   40   20
QUALITY, SCATTERS

FIGURE 11. - TOTAL COST QUANTIFIED IN TERMS OF QUALITY CONTROL
(GIVEN MEAN STRENGTH).

8

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15. Supplementary Notes

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16. Abstract

Because of the increasing need to account for the uncertainties in material properties, loading conditions, geometry, etc., a methodology has been developed to determine structural reliability and to assess the associated risk. The methodology consists of a probabilistic structural analysis by a probabilistic finite element computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) and a generic probabilistic material property model. The methodology is versatile and is equally applicable to structures operating at high- and cryogenic-temperature environments. Results obtained demonstrate that the issues of structural reliability and risk can be formally evaluated by using the methodology developed which is inclusive of the uncertainties in material properties, structural parameters, and loading conditions. The methodology is described in some detail with illustrative examples.

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